SM3 1.4: Synthetic Division and Remainder Thm Key

Vocabulary Problems: Interpret the results of the synthetic division problem by rewriting the problem as $\frac{dividend}{divisor} = quotient$.

1)
$$\begin{array}{r|ccc} \boxed{2} & 3 & 5 & 7 \\ & \downarrow & 6 & 22 \\ \hline & 3 & 11 & \boxed{29} \end{array}$$

$$\frac{3x^2+5x+7}{x-2} = 3x+11+\frac{29}{x-2}$$

2)
$$\begin{array}{r|ccc} \boxed{-7} & 1 & 2 & 4 \\ & \downarrow & -7 & 35 \\ \hline & 1 & -5 & \boxed{39} \end{array}$$

$$\frac{x^2+2x+4}{x+7} = x-5+\frac{39}{x+7}$$

3) What is the remainder of $\frac{4x-11}{x-6}$?

$$\begin{array}{r|cc} \boxed{6} & 4 & -11 \\ & \downarrow & 24 \\ \hline & 4 & \boxed{13} \end{array}$$

4) What is the remainder of $\frac{5x^3-2x^2+4x-9}{x+3}$?

$$\begin{array}{r|cccc} \boxed{-3} & 5 & -2 & 4 & -9 \\ & \downarrow & -15 & 51 & -165 \\ \hline & 5 & -17 & 55 & \boxed{-174} \end{array}$$

Problems: Simplify each expression into a quotient that contains a polynomial and $\frac{remainder}{divisor}$.

5) $\frac{x^2+10x+24}{x-2}$

$$\begin{array}{r|ccc} \boxed{2} & 1 & 10 & 24 \\ & \downarrow & 2 & 24 \\ \hline & 1 & 12 & \boxed{48} \end{array}$$

$$x+12+\frac{48}{x-2}$$

6) $\frac{x^2-3x+7}{x-4}$

$$\begin{array}{r|ccc} \boxed{4} & 1 & -3 & 7 \\ & \downarrow & 4 & 4 \\ \hline & 1 & 1 & \boxed{11} \end{array}$$

$$x+1+\frac{11}{x-4}$$

7) $\frac{x^2+7x-12}{x-3}$

$$\begin{array}{r|ccc} \boxed{3} & 1 & 7 & -12 \\ & \downarrow & 3 & 30 \\ \hline & 1 & 10 & \boxed{18} \end{array}$$

$$x+10+\frac{18}{x-3}$$

8) $\dfrac{x^2+x-30}{x+6}$

-6	1	1	-30
	$\downarrow$	-6	30
	1	-5	$\boxed{0}$

$x-5$

9) $\dfrac{x^2+11x+8}{x+1}$

-1	1	11	8
	$\downarrow$	-1	-10
	1	10	$\boxed{-2}$

$x+10+\dfrac{-2}{x+1}$

10) $\dfrac{x^2+4x-5}{x}$

0	1	4	-5
	$\downarrow$	0	0
	1	4	$\boxed{-5}$

$x+4+\dfrac{-5}{x}$

11) $\dfrac{3x^2+x+6}{x-8}$

8	3	1	6
	$\downarrow$	24	200
	3	25	$\boxed{206}$

$3x+25+\dfrac{206}{x-8}$

12) $\dfrac{-4x^2-3x-1}{x+6}$

-6	-4	-3	-1
	$\downarrow$	24	-126
	-4	21	$\boxed{-127}$

$-4x+21+\dfrac{-127}{x+6}$

13) $\dfrac{2x^2-12}{x-5}$

5	2	0	-12
	$\downarrow$	10	50
	2	10	$\boxed{38}$

$2x+10+\dfrac{38}{x-5}$

14) $\dfrac{x^3-5x^2+10x-50}{x-5}$

5	1	-5	10	-50
	$\downarrow$	5	0	50
	1	0	10	$\boxed{0}$

x^2+10

15) $\dfrac{x^3+x^2+x+1}{x-1}$

1	1	1	1	1
	$\downarrow$	1	2	3
	1	2	3	$\boxed{4}$

$x^2+2x+3+\dfrac{4}{x-1}$

16) $\dfrac{2x^3+3x^2+4x-5}{x+3}$

-3	2	3	4	-5
	$\downarrow$	-6	9	-39
	2	-3	13	$\boxed{-44}$

$2x^2-3x+13+\dfrac{-44}{x+3}$

17) $\dfrac{x^5 - 4x^4 - 2x^3 - 6x^2 + 9x + 3}{x - 2}$

$\boxed{2}$	1	-4	-2	-6	9	3
	$\downarrow$	2	-4	-12	-36	-54
	1	-2	-6	-18	-27	-51

$x^4 - 2x^3 - 6x^2 - 18x - 27 + \dfrac{-51}{x - 2}$

18) $\dfrac{7 - x^3 - 3x^4 + x^2 - 3x^5}{x + 3}$

$\boxed{-3}$	-3	-3	-1	1	0	7
	$\downarrow$	9	-18	57	-174	522
	-3	6	-19	58	-174	529

$-3x^4 + 6x^3 - 19x^2 + 58x - 174 + \dfrac{529}{x + 3}$

19) Prove whether $(x + 5)$ is a factor of $x^2 + 2x - 35$ and write a sentence explaining your reasoning.

$\boxed{-5}$	1	2	-35
	$\downarrow$	-5	15
	1	-3	$\boxed{-20}$

Because the remainder of $\frac{x^2+2x-35}{x+5}$ is not 0, $x + 5$ is not a factor of $x^2 + 2x - 35$.

20) Prove whether $(x - 2)$ is a factor of $x^3 - 3x^2 + 4$ and write a sentence explaining your reasoning.

$\boxed{2}$	1	-3	0	4
	$\downarrow$	2	-2	-4
	1	-1	-2	$\boxed{0}$

Because the remainder of $\frac{x^3-3x^2+4}{x-2}$ is 0, $x - 2$ is a factor of $x^3 - 3x^2 + 4$.

21) Given $f(x) = 4x^5 - 2x^3 + 17x^2 + 4$, find $f(-3)$.

$\boxed{-3}$	4	0	-2	17	0	4
	$\downarrow$	-12	36	-102	255	-765
	4	-12	34	-85	255	$\boxed{-761}$

Since the remainder is -761, then $f(-3) = -761$.

22) Given $f(x) = -7x^3 + 2x^2 + 15x - 26$, find $f(13)$.

$\boxed{13}$	-7	2	15	-26
	$\downarrow$	-91	-1157	-14846
	-7	-89	-1142	$\boxed{-14872}$

$f(13) = -14872$